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Repeat Offenders: If They Learn, We Punish Them More Severely*

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Abstract

Many legal systems are designed to punish repeat offenders more severely than first time offenders. However, existing economic literature generally offers either mixed or qualified results regarding optimal punishment of repeat offenders. This paper analyzes optimal punishment schemes in a two period model, where the social planner announces possibly-different sanctions for offenders based on their detection history. When offenders learn how to evade the detection mechanism employed by the government, escalating punishments can be optimal. The contributions of this paper can be listed as follows: First, it identifies and formalizes a source which may produce a marginal effect in the direction of punishing repeat offenders more severely, namely learning. Next, it identifies conditions under which the tendency in legal systems to punish repeat offenders more severely is justified. Overall, the findings suggest that the traditional variables identified so far in the literature are not the only relevant ones in deciding how repeat offenders should be punished, and that learning dynamics should also be taken into account.

Keywords: Repeat Offenders, Crime and Deterrence, Optimal Sanctions.

JEL classification: K00, K14, K42

I. Introduction and Literature Review

After Becker (1968) provided the framework to analyze crime and deterrence in the context of modern economic theory, many have engaged in the analysis of different dimensions of the problem.1 The existing literature on repeat offenders has generated different results, some of which directly contradict each other.2

The qualified nature of most results in the literature and the fact that there have been multiple answers to

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1 See Garoupa (1997) and Polinsky and Shavell (2000) for surveys of economic literature related to crime and deterrence.

2 See Emons (2003) and (2004) and Polinsky and Shavell (1998) for brief reviews on existing literature in economics dealing with the issue of repeat offenders.
the question of how repeat offenders should be punished, have lead some scholars to regard this problem as a "puzzle".3

This paper contributes to the interpretation of this puzzle, by identifying a source which may produce a marginal effect in the direction of punishing repeat offenders more severely. It introduces and formalizes the idea that repeat offenders may gain experience and learn from their offenses and convictions.4 This may lead offenders to be detected with a lower probability in their subsequent offenses. However, as pointed out in the existing literature, it is possible that law enforcers also learn.5 Once an individual gets a first conviction, he is known to the enforcers and this usually increases his probability of detection for a later crime, since law enforcers acquire information about that individual and may simply look for the 'usual suspects' first. The main result of this paper is that, if the net learning effect favors offenders rather than law enforcers, then there are optimal escalating punishment schemes.

Whether or not offenders learn more than law enforcers is an empirical question, and presumably it’s answer will depend on the type of offense. However, most likely there are offenses where the learning effect is greater on the offenders’ side. Consider the offenses of speeding, littering, riding on public transportation without a ticket or smoking in a non-smoking area. Most probably the learning effect on the law enforcers' side is close to none, whereas repeat offenders may very well learn about the mechanism employed by the law enforcers in detecting offenders. Hence, this paper provides a justification for the widely accepted practice of punishing repeat offenders more severely, at least for some types of offenses.

The plausibility of this learning assumption will become more clear after a brief review of the existing literature, which demonstrates the strict conditions and specialized assumptions required for escalating penalties to be optimal. Polinsky and Rubinfeld (1991), Polinsky and Shavell (1998), Chu et al. (2000), Miceli and Bucci (2005), and Emons (2007) are perhaps the most important articles which produce the result that some form of increasing penalties are optimal.

To demonstrate the restrictive conditions required for the derivation of optimal escalating penalties, quoting Miceli and Bucci (2005) is perhaps the ideal way to begin: "... this justification for escalating penalties, like earlier theories, seems to apply to a fairly restrictive set of circumstances—specifically, crimes that should definitely be deterred."6 Similar problems are encountered in other important articles as well.

Emons (2007) concludes that punishing repeat offenders more severely is optimal if the benefit from crime is sufficiently high. Even this qualified conclusion requires that individuals must be constrained to choose

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3 Dana (2001) and Emons (2003) explicitly refer to this problem as a puzzle. See Chu et al. (2000) for a discussion on the "qualified arguments" presented in other articles by Rubinstein (1979) and (1980), Polinsky and Rubinfeld (1991) and Burnovski and Safra (1994). The general tendency in the existing literature is to offer reasons through theoretical models as to why it may be optimal to punish repeat offenders more severely, but there are exceptions. For instance, Landsberger and Melijson (1982) are concerned with situations where different detection probabilities can be chosen for repeat offenders and first time offenders.

4 To the best of my knowledge, Polinsky and Rubinfeld (1991) is the first article which mentions a similar idea. See Polinsky and Rubinfeld (1991) Section 4.d. Dana (2001) also mentions this possibility.

5 See Garoupa and Jellal (2004) and Dana (2001), for a discussion on the impacts of learning by the government which Garoupa and Jellal refer to as the "demand side".

6 To justify escalating penalties, they also assume that within society there are irrational people who commit crime regardless of the severity of punishments.
between either always intentionally committing crime or never intentionally committing crime.\textsuperscript{7} Hence, this result is dependent on a specialized assumption, one requiring individuals to commit to a certain path of actions.\textsuperscript{8} Chu et al. (2000) derive a limited result, namely that increasing penalty schemes are better than uniform ones. They do not compare increasing penalty schemes to decreasing ones. Furthermore, social welfare is not the sum of individuals’ utilities and it is assumed that gains from crime are illicit. Polinsky and Shavell (1998) is perhaps less restrictive. However, their result is not consistent with the widely accepted legal practice of punishing repeat offenders more severely. It states that, for his second offense, a person should be punished the same way he was punished for his first offense.\textsuperscript{9} This does not describe the widely employed legal practice of punishing repeat offenders more severely. Polinsky and Rubinfeld (1991) assume that individuals’ benefits are composed of acceptable and illicit gains. This assumptions is referred to as ‘nonstandard’ in the literature and is interpreted as being problematic.\textsuperscript{10} Furthermore, their main result states that escalating penalties can be optimal only within some parametric range.\textsuperscript{11}

In contrast to the articles which I have briefly reviewed, the instant paper does not assume (i) that individuals may have illicit benefits, (ii) that there are irrational individuals, or (iii) that individuals must commit to a certain path of actions. Furthermore, it derives the objective of the legal system by aggregating individuals’ utilities as opposed to simply assuming certain cost functions and it does not eliminate decreasing penalty schemes as available policy tools. It could therefore be stated that the only nonstandard assumption in this model is the one on learning and accordingly that the model’s deviation from the standard set of assumptions in the crime and deterrence literature is quite small. Finally, this paper is different from Polinsky and Shavell (1998) in that it justifies the current legal practice, at least for a certain class of crimes.

It should also be noted that the learning assumption in this paper is distinct from those introduced in Sah (1991), Ben-Shahar (1997), and At and Chappe (2008). These models assume individuals lack information about the frequency with which they will be caught if they commit an offense, whereas the instant model assumes that individuals gain experience in breaking the law, and therefore evade detection more often. Furthermore, the focus of and the results obtained in these papers are not related to optimal punishment schemes when repeated offenses are possible.

To distinguish the learning assumption employed in the instant paper from earlier learning assumptions, consider Ben-Shahar (1997). Assume the actual probability of detection is $\frac{1}{2}$ and the actual sanction is $\$100$, making the expected sanction $\$50$. Some individuals have inaccurate beliefs about the expected sanction. In particular, there can be an individual, call him ‘\textit{Schopenhauer’}, who thinks that the expected sanction is $\$150$. If \textit{Schopenhauer} is convicted of a crime, he ‘learns’ that the actual probability of detection is $\frac{1}{2}$ and

\textsuperscript{7}In other words, the individuals’ strategies are constrained to be history independent.

\textsuperscript{8}This specialized assumption is not employed in Emons (2003) or (2004), and those papers imply that escalating penalties are never optimal. This should also demonstrate that results in Emons (2007) are dependant on this specialized assumption. Another non-traditional assumption employed in Emons (2007) is that gains from crime are illicit.

\textsuperscript{9}See Proposition 1 in Polinsky and Shavell (1998).

\textsuperscript{10}See Polinsky and Shavell (1998) footnote 16. See also Dana (2001) Section II. D. 1. for a discussion concerning the problems associated with this assumption.

\textsuperscript{11}It has been noted in Chu et al. (2000) Section 1.2., that this result could be more useful, if it could be shown that this particular parametric range is consistent with reality.
the actual sanction is $50. In contrast, in my model, Schopenhauer knows that the actual probability of detection is $\frac{1}{2}$ and that the actual sanction is $50$, from the beginning. He does not need to learn this through the commission of a crime. However, if he commits crime, his probability of detection will be different in the second period, for instance $\frac{1}{4}$. In this case, Schopenhauer has learned how to evade detection as opposed to learning about how frequently detection takes place.

In sum, this paper formalizes a particular type of learning for the first time in the law and economics literature. It shows that this innovative assumption is sufficient to justify the practice of punishing repeat offenders more severely, when the net learning effect favors offenders. Accordingly, reliance on other types of nonstandard assumptions, which were used in previous important articles deriving optimality conditions for escalating penalties, are not necessary.

The rest of this paper is organized as follows: Section II provides a short description of the model, derives individuals’ best responses to sanctions and a pure utilitarian social welfare function, and states the general result, Section III is devoted to remarks and conclusions. An appendix at the end contains the proof of a theorem.

II. The Model

Society consists of individuals who are continuously distributed over benefits ($b$) from committing an act which causes an expected harm of $h$ to society. There are two periods in which individuals decide whether or not to commit this act, which henceforth will be referred to as an offense. Individuals’ benefits are not observable by the government. The government invests in a detection mechanism, which catches offenders with a certain probability. This probability ($p$) is assumed to be fixed and is interior.\footnote{This is a commonly employed assumption in the literature. Polinsky and Rubinfeld (1991), Burnovski and Safra (1994), Chu et. al. (2000), Nyborg and Telle (2004) and Miceli and Bucci (2005) are examples of models which impose this assumption. Furthermore, when general enforcement is possible, for low levels of harm, $p$ can be treated as a fixed value although it is endogenously determined. The last point is formalized in Shavell (1991) and its implications are discussed in Section III.}

There are three types of individuals in this model: First time offenders (Type $F$), Offenders with a clean record (Type $O$), and ex-Convicts (Type $C$). In the first period all individuals are Type $F$, and choose whether or not to commit the offense. Those who choose not to commit the offense, remain Type $F$ individuals. Those who commit the offense but are not detected become Type $O$ individuals, whereas those who are detected become Type $C$ individuals.

Since the government may only observe the detection history of individuals, it cannot distinguish between Type $F$ and $O$ individuals. Therefore, Type $F$ and $O$ individuals will collectively be referred to as non-detected individuals (Type $N$). The government announces (possibly different) sanctions for ex-convicts (Type $C$) and individuals with a clean record (Type $N$).

This model allows for learning: Offenders (Types $O$ and $C$) learn about the way in which the detection mechanism works. The model also allows for additional learning from conviction: Type $C$ individuals may learn more than Type $O$ individuals. Formally, let $q_{O}$ and $q_{C}$ refer respectively to the probabilities with
which non-detected offenders and ex-convicts are detected in the second period if they decide to commit an offense. The probability of detection for first time offenders is still $p$. These probabilities describe learning effects and it follows that at the absence of learning on the law enforcer’s side they have the following relation: $q'_C \leq q_O < p$. The learning effect from the commission of an offense, which occurs purely due to experience is captured by $p - q_O = E$. The learning effect from being convicted, is captured by $q_O - q'_C = X$.

However, law enforcers may also learn. It is assumed that law enforcers will be able to detect ex-convicts easier, because they have information about these individuals in their records. This advantage, or learning effect will be captured by the parameter $L$, which will increase the probability of detection of ex-convicts ($q'_C$). Hence, the adjusted probability of detection for ex-convicts ($q_C$), which is the relevant one for the analysis, will be higher than $q'_C$ (since $q_C = q'_C + L > q'_C$). Furthermore, it does not follow that $q_C \leq q_O < p$.

The next sections will derive optimality conditions depending on the relationship between $L$, $X$ and $E$.

Other assumptions employed in the model can be listed as follows: Individuals are risk-neutral expected utility maximizers with additive utility over time and they do not have an inability to pay high fines. Sanctions and probabilities announced by the government are common knowledge and are credible. Collection of fines is costless.

The following list provides the notation, and formalizes some assumptions.

**Notation**

$b \in [0, \infty)$; type of individual denoting benefits received from committing the offense.

$h > 0$; harm generated by the offense.

$p$, $q_O$, $q_C$; probabilities that Type $F$, $O$ and $C$ individuals are detected respectively.

$q'_C$; probability that an ex-convict would be detected if there was no learning on the law enforcer’s side.

$X$, $E$; magnitudes of learning from being convicted and the commission of an offense respectively.

$L$; magnitude of learning by law enforcers.

$s_N > 0$; finite monetary fine extracted from non-detected individuals.

$s_C > 0$; finite monetary fine extracted from ex-convicts.

$(; ;)$; Defines a sanction pair, where the first and second components refer to $s_N$ and $s_C$ respectively.

**Individuals’ Decision Making Process**

In this sub-section, individuals’ best responses to sanctions chosen by the government are analyzed in the standard way.

When the second period is reached, individuals need not consider anything but the second period payoffs associated with their decisions, hence an immediate observation can be made as follows:

**Observation 1** (i) A type $F$ individual, commits offense in the second period iff $b > ps_N$. (ii) A type $O$ individual, commits offense in the second period iff $b > q_O s_N$. (iii) A type $C$ individual, commits offense in the second period iff $b > q_C s_C$.

\footnote{The no-maximal fine assumption is mainly a simplifying one. Remarks concerning the no-maximal fine and fixed probability of detection assumptions is provided in Section III.}
These inequalities govern the decisions of all individuals in the second period. For \(i \in \{F, O, C\},\) let \(\Pi_{i,2}\) denote the second period expected benefit of a Type \(i\) individual in the second period. Utilizing Observation 1 leads to a next observation which will be useful in describing individuals’ behavior in the first period:

**Observation 2** \(\text{Individuals’ payoff in the second period, depending on their types will be given by:}\)

\[
\Pi_{F,2} = \max\{b - ps_N, 0\}, \quad \Pi_{O,2} = \max\{b - q_Os_N, 0\}, \quad \Pi_{C,2} = \max\{b - q_Cs_C, 0\}.
\]

Individuals foresee the way they will behave in the second period, and that this behavior will depend on whether or not they are caught in the first period. Accordingly, let \(\Pi_{c,1}\) and \(\Pi_{n,1}\) respectively denote the expected utilities from committing the offense and abstaining from it in the first period. These are given by:

\[
\Pi_{c,1}(b) = [b - ps_N] + [p\Pi_{C,2}(b) + (1 - p)\Pi_{O,2}(b)] \quad \text{and} \quad \Pi_{n,1}(b) = \Pi_{F,2}(b)
\]

An individual with benefit \(b\) will commit the offense in the first period iff \(\Pi_{c,1}(b) \geq \Pi_{n,1}(b)\). By utilizing (1), this inequality becomes:

\[
[b - ps_N] + [p\Pi_{C,2}(b) + (1 - p)\Pi_{O,2}(b)] \geq \Pi_{F,2}(b)
\]

Observation 1 and inequality (2) describe how individuals will react to sanction pairs announced by the government. Knowing how individuals will react to these, the government may announce sanction pairs to achieve optimality.

**Social Welfare and Policy Implications**

A pure utilitarian social welfare function is used to evaluate the desirability of outcomes. Since individuals are assumed to be risk neutral expected utility maximizers, and since revenues and harms will be distributed back to society, the utilitarian social welfare function is simply the sum of all individuals’ benefits from violating the law minus aggregate harms:

\[
\int_{\mu_1(s_N, s_C)} (b - h) f(b) db + \int_{\mu_2(s_N, s_C)} (b - h) f(b) db
\]

where \(\mu_1(s_N, s_C)\) and \(\mu_2(s_N, s_C)\) denote respectively the set of individuals who decide to commit the offense in the first and second periods as a function of the sanctions announced by the government and \(f\) is the density function describing the distribution of individuals over benefits.

Social welfare is obviously maximized if \(s_N\) and \(s_C\) can be chosen such that the set of individuals who commit the offense are those individuals with \(b > h\). This condition will be referred to as first-best deterrence. Proposition 1 identifies the necessary and sufficient conditions to achieve first-best deterrence.

**Proposition 1:** First-best deterrence is achieved iff the following constraints simultaneously hold:

\[
q_Os_N \leq h
\]

\[
q_Cs_C \leq h
\]

\[
2h - s_N((1 - p)q_O + p) - s_C[pq_C] = 0
\]
Proof: Assuming the sanction pair announced by the government induces first-best deterrence, there will be individuals of Type $O$ and $C$ in the second period with benefits $b > h$. First-best deterrence requires that these individuals are not deterred from committing the offense in the second period, hence by Observation 1, $s_N$ and $s_C$ must be such that (4) and (5) hold. Next, observe that $ps_N \geq h$ must hold for first-best deterrence, otherwise there will be individuals with $b < h$ who commit the offense in the first period.¹⁴ This implies that $\Pi_{n,1}(h) = 0$. Furthermore, note that $\Pi_{c,1}$ is strictly increasing in $b$ and at least as fast as $\Pi_{n,1}$. Hence, setting $\Pi_{c,1}(h) = 0$ will guarantee that only individuals with $b > h$ will commit the offense. Given that (4) and (5) hold, $\Pi_{c,1}(h) = [h - ps_N] + [p(h - qCS_C) + (1 - p)(h - qOS_N)]$. Setting this expression equal to zero, we have (6).■

Proposition 1 identifies the conditions under which first-best deterrence is possible. Depending on the learning parameters ($E, X$ and $L$), first-best deterrence may or may not be achievable through increasing penalty schemes. This is summarized by the following:

Theorem 1: (i) If offenders learn more in comparison to law enforcers ($\frac{E}{2-p} + X > L$), then there is always an optimal increasing penalty scheme. (ii) Otherwise ($\frac{E}{2-p} + X \leq L$), there are no increasing penalty schemes which induce first-best deterrence.

Proof: See Appendix.

Theorem 1 summarizes when it is optimal to punish repeat offenders more severely. The main statement of the theorem is self explanatory. Perhaps a short remark should be made to highlight a property associated with this result. The condition for optimal increasing penalty schemes, namely $\frac{E}{2-p} + X > L$, is an asymmetric one. This is so, because offenders have two separate ways from which they can learn to evade the detection mechanism, whereas law enforcers have a single way to gather information about individuals. Hence, even if offenders are put in a disadvantage by being convicted (if $L - X > 0$), increasing penalty schemes may still be optimal (i.e. when $\frac{E}{2-p} > L - X > 0$).

The next section provides a few concluding remarks concerning some assumptions employed in the derivation of Theorem 1 and a few remarks concerning types of crimes for which the learning condition in Theorem 1 is likely to be satisfied.

III. Remarks and Conclusion

It has been shown that the practice of punishing repeat offenders more severely is justifiable, when there is a learning effect on the offenders’ side which outweighs the learning effect on the law enforcer’s side. The main idea driving this result is simple. When offenders are talented in evading the law, they have to be punished more severely so that optimal deterrence can be achieved. Interpreting this result, requires a closer look at the simplifying assumptions invoked.

The two main simplifying assumptions were that the probability of detection, $p$, is fixed, and that

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¹⁴(5) and (6) together imply that $ps_N \geq h$, hence it is not necessary to state this constraint separately.
individuals do not have a binding wealth constraint. These two assumptions make first-best deterrence desirable. The types of crimes for which these assumptions are less problematic can be identified by taking a closer look at Shavell (1991). In this article, the author shows that for crimes which do not result in great harms, general enforcement effort by law enforcers should not be complemented by specific enforcement, and that individuals’ wealth constraints are non-binding. Hence, for crimes which do not result in great harms, these assumptions seem to be plausible.

More importantly, the same types of crimes seem to be those in which the learning effect on the offenders’ side are more likely to outweigh the learning effect on the law enforcers’ side. This follows from a similar reasoning as in Shavell (1991). Although not explicitly modelled in this paper or in Shavell (1991), learning on the enforcers’ side is presumably costly, and therefore can be interpreted as a special type of specific enforcement effort. If it is not worthwhile for law enforcers’ to complement general enforcement effort by specific enforcement, it is most likely not worthwhile to complement it by engaging in efforts which would lead to learning on the law enforcers’ side either.

This conjecture seems to be verified in reality. Offenses such as speeding, double parking, smoking in a non-smoking area, littering and riding on public transportation without a ticket, result in harms which would not justify the cost of learning on the law enforcers’ side and are usually enforced through general enforcement efforts only. This being the case, the probability of detection, \( p \), is fixed at a certain optimal level for all types of crimes which are enforced through the same general enforcement mechanism. Furthermore, since the relative level of harms for such crimes are low, wealth constraints do not become an issue. Due to these reasons, it would be safe to assert that the results obtained in this paper provide a justification for severer punishments for repeat offenders when the crime in question results in small harms to society.

The analysis provided in this paper may also guide us in interpreting the relative importance of learning by offenders versus law enforcers. Repeat offenders are punished more severely in most legal systems. Absent of other considerations, and assuming that this practice is in-fact an efficient one, one can conclude by using a positive approach that learning by offenders is perhaps more important than learning by law enforcers in many cases. Given that this is the case, the next question is whether different types of crimes should be treated differently in terms of increasing penalties for recidivism. Based on the analysis provided in this paper, this question is answered positively. The increase in the punishment for a recidivist should depend on the typical learning effects associated with the crime in question. In particular, the higher the net learning effect is on the offenders’ side, the higher should be the increase in the punishment of a recidivist. Therefore, a deeper understanding and a rigorous analysis of issues related to learning are of great importance.

The ideas formalized in this paper also have important implications concerning procedural issues in criminal law, in particular issues concerning the standard of proof. If repeat offenders become more sophisticated, through the commission of offenses, it becomes exceedingly harder to prove a non-detected offender’s guilt beyond a reasonable doubt. In this case, one might argue that the optimal standard of proof should be
lower than it ordinarily would be, or that aggregating probabilities of guilt across criminal cases should be possible.\footnote{The latter point is made by Harel and Porat (2008) even at the absence of learning, but as they acknowledge, their argument seems to be stronger when offenders learn as in this paper.} This is another reason as to why issues related to learning should be investigated with care in future research.

In sum, the contributions of this paper can be listed as follows: First, it identifies and formalizes a source which produces a marginal effect in the direction of punishing repeat offenders more severely. Second, under specific assumptions, it justifies the tendency in legal systems to punish recidivists more severely. Overall, the findings in this paper suggest that the traditional variables identified so far in the literature (harms, benefit distributions, costs of detection, etc.) are not the only relevant ones in deciding how repeat offenders should be punished. The degree of learning by offenders and law enforcers is also a very important factor which should influence the way punishment schemes and perhaps criminal procedures are designed.

### Appendix

**Proof of Theorem 1:**

Part (i): Choose $s_C = \frac{h}{q_C}$. This satisfies (5). To satisfy (6) simultaneously, (A1) below must hold.

$$2h - s_N[(1-p)q_O + p] - hp = 0 \iff s_N = \frac{(2 - p)}{[(1-p)q_O + p]}h$$  \hspace{1cm} (A1)

This value of $s_N$ satisfies (4). Hence, $s^* = (\frac{(2 - p)h}{[(1-p)q_O + p]}; \frac{h}{q_C})$ satisfies (4), (5) and (6) making it an optimal sanction pair. Furthermore,

$$s_C = \frac{h}{q_C} > \frac{(2 - p)h}{[(1-p)q_O + p]} = s_N \iff \frac{E}{(2 - p)} + X > L$$  \hspace{1cm} (A2)

This implies that $(\frac{(2 - p)h}{[(1-p)q_O + p]}; \frac{h}{q_C})$ is an increasing sanction scheme when $\frac{E}{(2 - p)} + X > L$.

Part (ii): (6) implicitly defines $s_C$ as a decreasing function in $s_N$. Furthermore, $s^*$ is that sanction pair with the highest $s_C$ satisfying (5). Hence, $s^*$ as defined in Part (i) is the sanction pair with the smallest $\frac{s_N}{s_C}$ ratio, which satisfies (6) and (5) together. Therefore, if $s^*$ is not an increasing sanction pair, there are no increasing sanction pairs satisfying (5) and (6) together. But when $\frac{E}{(2 - p)} + X \leq L$, (A2) implies that $s^*$ is not an increasing sanction pair and accordingly there is no increasing sanction pair which achieves first-best deterrence.$\blacksquare$

### References


